Midterm 1 Practice Problems

ANSWER KEY

Competitive Balance

1 A hockey club won five straight titles. Its home attendance fell from 900,000 in Year 1 to 702,000 in Year 5, while a rival league with different champions grew from 8,000,000 to 9,200,000. Calculate the percent change for each.

Answer

$$\text{Percent change} = \frac{\text{Year 5} - \text{Year 1}}{\text{Year 1}}$$

$$\text{Dynasty:} \quad \frac{702{,}000 - 900{,}000}{900{,}000} = \frac{-198{,}000}{900{,}000} = -0.22 = -22\%$$

$$\text{Rival:} \quad \frac{9{,}200{,}000 - 8{,}000{,}000}{8{,}000{,}000} = \frac{1{,}200{,}000}{8{,}000{,}000} = 0.15 = +15\%$$

2 In a two-team league, $MR_L = 4.0 - 0.05W$, $MR_S = 3.0 - 0.05W$, and $MC = 1.0$ for both ($ millions per point of win %). Find each club's profit-maximizing win percentage. If both start at .500, what does each club do next?

Answer

Set $MR = MC$ for each team.

$$\text{Large:} \quad 4.00 - 0.05W = 1 \implies 3.00 = 0.05W \implies W = \tfrac{3.00}{0.05} = 60$$

$$\text{Small:} \quad 3.00 - 0.05W = 1 \implies 2.00 = 0.05W \implies W = \tfrac{2.00}{0.05} = 40$$

Check: $60 + 40 = 100$, which is consistent with a two-team league.

At .500 ($W = 50$):

$$MR_L = 4.0 - 0.05(50) = 1.5 > MC = 1.0 \qquad MR_S = 3.0 - 0.05(50) = 0.5 < MC = 1.0$$

The large-market team's next win is worth more than it costs, so it buys talent to increase its win percentage. The small-market team's next win is worth less than it costs, so it sells talent to decrease its win percentage.

3 A six-team league plays a 20-game season. Final win totals are 15, 12, 11, 9, 8, and 5. Calculate the standard deviation of winning percentage σ_w, the idealized standard deviation $\sigma_{ID} = 0.5/\sqrt{G}$, and the ratio $R = \sigma_w/\sigma_{ID}$.

Answer

Step 1: Winning percentages and squared deviations from .500.

Wins	**WPCT** $=$ Wins$/20$	**WPCT** $-$ **.500**	**(WPCT** $-$ **.500)**^{2}
15	.75	.25	.0625
12	.60	.10	.0100
11	.55	.05	.0025
9	.45	$-.05$	.0025
8	.40	$-.10$	.0100
5	.25	$-.25$	.0625
		Sum	**.1500**

Step 2: Standard deviations and ratio.

$$\sigma_w = \sqrt{\frac{.150}{6}} = \sqrt{.025} = .158$$
$$\sigma_{ID} = \frac{.5}{\sqrt{20}} = \frac{.5}{4.472} = .112$$
$$R = \frac{\sigma_w}{\sigma_{ID}} = \frac{.158}{.112} = 1.41$$

Since $R > 1$, the league is less balanced than a league of equally good teams.

4 A team's winning percentages over five seasons are .650, .500, .700, .550, and .600. Calculate its between-season standard deviation.

Answer

Step 1: The team's average. Between seasons, we subtract the team's own average, not .500:

$$\overline{\text{WPCT}} = \frac{.65 + .50 + .70 + .55 + .60}{5} = \frac{3.00}{5} = .60$$

Step 2: Squared deviations.

$$(.65-.6)^2+(.50-.6)^2+(.70-.6)^2+(.55-.6)^2+(.60-.6)^2 = .0025+.01+.01+.0025+0 = .025$$

Step 3: Standard deviation.

$$\sigma_w = \sqrt{\frac{.025}{5}} = \sqrt{.005} = .071$$

5 Over 20 seasons, a league's titles were won as follows: A: 7, B: 4, C: 3, D: 2, E: 2, F: 1, G: 1. Calculate the 1-firm and 3-firm concentration ratios, the HHI, and the minimum possible HHI.

Answer

Shares of titles: A $= 7/20 = .35$, B $= .20$, C $= .15$, D $= .10$, E $= .10$, F $= .05$, G $= .05$

$$\begin{aligned}
\text{1-firm} &= .35 \\
\text{3-firm} &= \frac{7+4+3}{20} = .35 + .20 + .15 = .70 \\
\text{HHI} &= .35^2 + .20^2 + .15^2 + .10^2 + .10^2 + .05^2 + .05^2 \\
&= .1225 + .04 + .0225 + .01 + .01 + .0025 + .0025 = .21 \\
&\left(\text{equivalently } \tfrac{7^2+4^2+3^2+2^2+2^2+1^2+1^2}{20^2} = \tfrac{84}{400} = .21\right)
\end{aligned}$$

Minimum HHI: a different champion every season, so 20 teams each with a share of $1/20$: $20 \times \left(\frac{1}{20}\right)^2 = \frac{1}{20} = .05$

6 The competitive balance tax threshold is \$241 million, with rates of 20% (first year over), 30% (second straight year), and 50% (third straight year or later). A club's payroll is \$280 million in its third straight year over. Calculate the tax.

Answer

Amount over the threshold: $\$280M - \$241M = \$39M$
Third straight year, so the rate is 50%: $\$39M \times .50 = \$19.5M$

Sports Gambling

7 The home team has a 60% chance of winning. Find the fair decimal, fractional, and moneyline odds for each team, and show that a \$1 bet on the underdog at fair decimal odds has $EV = \$1$.

Answer

$p_H = .60$ and $p_A = 1 - .60 = .40$

Decimal $= 1/p$: Home $= 1/.60 = 1.67$, Away $= 1/.40 = 2.50$
Fractional $=$ Decimal -1: Home $= 1.67 - 1 = 0.67 = 2/3$, Away $= 2.50 - 1 = 1.50 = 3/2$
Moneyline: the favorite is -100/fractional, the underdog is $+100 \times$ fractional:
Home $= -100/(2/3) = -150$, Away $= +100 \times (3/2) = +150$

Expected value of \$1 on the underdog: if Away wins (prob. .40) you get back \$2.50; if it loses (prob. .60) you get back \$0.

$$EV_{\text{Away}} = .40(\$2.50) + .60(\$0) = \$1.00$$

You expect to get back exactly the \$1 you bet, so the bet is fair.

8 The favorite truly wins 70% of the time. A sportsbook posts favorite -250 and underdog $+200$. Convert each line to an implied probability and add them. Then calculate the expected value of a \$100 bet on each team. Which bet is worse?

Answer

Implied probabilities. A negative line uses $\dfrac{|ML|}{|ML|+100}$; a positive line uses $\dfrac{100}{ML+100}$.

$$\text{Favorite: } \frac{250}{250+100} = \frac{250}{350} = 71.4\% \qquad \text{Underdog: } \frac{100}{200+100} = \frac{100}{300} = 33.3\%$$

$71.4\% + 33.3\% = 104.8\%$. With fair odds these would add to exactly 100%; the extra 4.8% is the vig.

Profit on a winning \$100 bet. At -250 you must bet \$250 to win \$100, so a \$100 bet wins $\$100 \times \frac{100}{250} = \$40$. At $+200$, a \$100 bet wins \$200. A losing bet loses the \$100.

Expected value (true probabilities: favorite .70, underdog .30):

$$EV_{\text{Fav}} = .70(\$40) - .30(\$100) = \$28 - \$30 = -\$2$$
$$EV_{\text{Dog}} = .30(\$200) - .70(\$100) = \$60 - \$70 = -\$10$$

Both bets lose money on average, but the underdog bet is worse.

9 A spread bet is offered at -110 on both sides, and \$143,000 is bet on each side. How much does the sportsbook pay out, how much does it keep, and what is its cut as a share of the handle?

Answer

Handle (total wagered): $\$143{,}000 \times 2 = \$286{,}000$

Payout to the winning side. At -110, each \$110 bet wins \$100, and winners also get their stakes back:

$$\text{Winnings} = \$143{,}000 \times \tfrac{100}{110} = \$130{,}000$$
$$\text{Payout} = \$130{,}000 + \$143{,}000 = \$273{,}000$$

Kept: $\$286{,}000 - \$273{,}000 = \$13{,}000$

Cut: $\dfrac{\$13{,}000}{\$286{,}000} = .045 = 4.5\%$ of the handle

10 Calculate the break-even win rate for a bet at -115.

Answer

At -115 you risk \$115 to win \$100. Break even when expected profit is zero:

$$p(100) - (1-p)(115) = 0 \implies 100p = 115 - 115p \implies 215p = 115 \implies p = \frac{115}{215} = 53.5\%$$

11 $4,000 is bet to win on five horses, and the track takes 20%. Calculate the prize pool, then each horse's implied win % and decimal odds. The track rounds payouts down to the nearest $0.10 per $1. Calculate the breakage if Horse 1 wins.

Horse	**1**	**2**	**3**	**4**	**5**
Handle	$1,500	$1,100	$750	$400	$250
Implied win %	37.50	27.50	18.75	10.00	6.25
Decimal odds	2.133	2.909	4.267	8.000	12.800

Answer

Prize pool: the track keeps 20%, so $\$4{,}000 \times .80 = \$3{,}200$

Implied win % $= \dfrac{\text{handle on horse}}{\text{total handle}}$: $\frac{1{,}500}{4{,}000} = 37.50\%$, $\frac{1{,}100}{4{,}000} = 27.50\%$, $\frac{750}{4{,}000} = 18.75\%$, $\frac{400}{4{,}000} = 10.00\%$, $\frac{250}{4{,}000} = 6.25\%$

Decimal odds $= \dfrac{\text{pool}}{\text{handle on horse}}$: $\frac{3{,}200}{1{,}500} = 2.133$, $\frac{3{,}200}{1{,}100} = 2.909$, $\frac{3{,}200}{750} = 4.267$, $\frac{3{,}200}{400} = 8.000$, $\frac{3{,}200}{250} = 12.800$

Breakage if Horse 1 wins. The track rounds $2.133 down to $2.10 per $1 bet:

$$\text{Paid out} = \$2.10 \times 1{,}500 = \$3{,}150$$
$$\text{Breakage} = \text{Pool} - \text{Paid out} = \$3{,}200 - \$3{,}150 = \$50$$

12 A minor-league umpire earns $60,000 a year with 20 years left and a 5% chance of being caught. If caught, they lose all remaining career earnings. Using $B > \Pr(C) \times C$, would they accept a $75,000 bribe? What is the smallest bribe that makes cheating rational?

Answer

Cost if caught: $C = \$60{,}000 \times 20 = \$1{,}200{,}000$
Expected cost: $\Pr(C) \times C = .05 \times \$1{,}200{,}000 = \$60{,}000$
Compare: $B = \$75{,}000 > \$60{,}000$, so yes, cheating is "rational" and the umpire would accept the bribe.
Smallest bribe: any bribe greater than $60,000 makes cheating rational.

13 A resident moves $25 a week from lottery tickets, where the state keeps 35% of spending, to sports betting, where the sportsbook keeps 9% of the handle and the state taxes that revenue at 15%. Calculate the state's annual revenue from this person before and after the switch.

Answer

Annual spending: $\$25 \times 52 = \$1{,}300$
Before (lottery): $\$1{,}300 \times .35 = \$455$
After (sports betting):
Sportsbook revenue: $\$1{,}300 \times .09 = \$117$
State tax on that revenue: $\$117 \times .15 = \$17.55$
The state's revenue from this person falls by $\$455 - \$17.55 = \$437.45$ a year.

Stadiums and Public Finance

14 A team offers to pay a city \$15 million a year for 4 years, starting one year from today, if the city builds it an arena. The interest rate is 8%. Calculate the present value of the payments and the city's NPV if construction costs \$45 million.

Answer

Discount each payment by $(1.08)^t$, where t is the year it is received:

Year t	**Payment** R	$\mathbf{(1.08)^t}$	**PV** $= R/(1.08)^t$
1	\$15M	1.0800	$15/1.0800 = \$13.89$M
2	\$15M	1.1664	$15/1.1664 = \$12.86$M
3	\$15M	1.2597	$15/1.2597 = \$11.91$M
4	\$15M	1.3605	$15/1.3605 = \$11.03$M
Total	\$60M		**\$49.68M**

$$NPV = PV - \text{Cost} = \$49.68M - \$45M = \$4.68M$$

The \$60M in payments is worth only \$49.68M today, but that is still more than the \$45M cost.

15 A team offers to pay a city \$8 million in Year 1, \$10 million in Year 2, \$12 million in Year 3, and \$15 million in Year 4 if the city builds it an arena. The interest rate is 6%, and construction costs \$38 million, paid today. Using

$$PV = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_T}{(1+r)^T}, \qquad NPV = PV - \text{Cost},$$

complete the table and calculate the city's NPV. Should the city build?

Year	**Payment**	$\mathbf{(1.06)^t}$	**PV**
1	\$8M	1.0600	$8/1.0600 = \$7.55$M
2	\$10M	1.1236	$10/1.1236 = \$8.90$M
3	\$12M	1.1910	$12/1.1910 = \$10.08$M
4	\$15M	1.2625	$15/1.2625 = \$11.88$M
Total	\$45M		**\$38.40M**

Answer

$$PV = 7.55 + 8.90 + 10.08 + 11.88 = \$38.40M \qquad NPV = \$38.40M - \$38M = \$0.40M$$

Yes, the city should build, since $NPV > 0$: the present value of the payments is greater than the cost.

16 A team's NPV from staying in its current stadium is \$150 million. A new stadium would generate revenue with a present value of \$1.1 billion and cost \$1.0 billion. What does the team do if it pays the full cost? What is the smallest public contribution that makes it prefer to build?

Answer

Paying the full cost: $NPV_{\text{new}} = \$1.1B - \$1.0B = \$100M$
Since $\$100M < \$150M$, the team stays in its current stadium.

Smallest public contribution S: the team builds if the new stadium's NPV is at least as large as staying:

$$\$1.1B - (\$1.0B - S) \geq \$150M \implies \$100M + S \geq \$150M \implies S \geq \$50M$$

The city would need to contribute at least \$50M.

17 A team draws 1,000,000 fans a year, each spending \$85; 75% are locals who would have spent the money in the city anyway. With $MPC = 0.75$ and $MPI = 0.35$, calculate the new spending, the open-economy multiplier $M = 1/(1 - MPC + MPI)$, and the total impact.

Answer

Total spending: $1{,}000{,}000 \times \$85 = \$85M$
New spending: only the 25% from non-locals is new to the city: $.25 \times \$85M = \$21.25M$
Multiplier: $M = \dfrac{1}{1 - .75 + .35} = \dfrac{1}{.60} = 1.67$
Total impact: $\$21.25M \times 1.67 = \$35.4M$

18 A city adds a \$6-a-night hotel tax. Before the tax, 15 million stays are sold at \$120. After, tourists pay \$124, hotels receive \$118, and stays fall to 13 million. How much did the city expect tourists to pay on the original stays? How much do tourists and hotels actually pay?

Answer

Expected: $\$6 \times 15M = \$90M$, all paid by tourists
Tourists: price rises from \$120 to \$124, so they pay \$4 of each night's tax: $\$4 \times 13M = \$52M$
Hotels: they now keep \$118 instead of \$120, so they pay \$2 of each night's tax: $\$2 \times 13M = \$26M$
Total collected: $\$52M + \$26M = \$78M = \$6 \times 13M$, less than the \$90M expected because stays fell.

19 A corporate bond pays 7% and its interest is taxed at 35%; a municipal bond's interest is tax-free. What municipal rate leaves the investor equally well off? How much does a city save each year borrowing \$500 million at that rate instead of 7%?

Answer

After-tax corporate return: $7\% \times (1 - .35) = 7\% \times .65 = 4.55\%$
So a tax-free municipal bond paying 4.55% leaves the investor equally well off.

City's savings: interest at 7% is $\$500M \times .07 = \$35M$; at 4.55% it is $\$500M \times .0455 = \$22.75M$.
Savings $= \$35M - \$22.75M = \$12.25M$ per year.