Consumer and Producer Theory Quick Notes

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Consumer Maximization Problem

- $\max_{x} u(x)$ subject to $px \leq w$, $x_k \geq 0$, $1 \leq k \leq m$
- Budget set: $B(p, w) \equiv \{x \mid px \leq w\}$
- A solution exists if $u(\cdot)$ is continuous and $B(p, w)$ is compact (Weierstrass)

Walrasian Demand Correspondence

- $x(p, w) = \arg\max_{x \in B(p,w)} u(x)$

Walras Law

- If $X = \mathbb{R}^m_+$ and $\succeq$ is continuous and locally nonsatiated, then $x(p, w)$ satisfies:
  1. Homogeneous of degree 0: $x(\alpha p, \alpha w) = x(p, w)$
  2. Budget exhaustion: $px = w$
  3. Upper hemicontinuity
  4. If $\succeq$ is convex, then $x(p, w)$ is convex-valued. If strictly convex, $x(p, w)$ is a function

Substitution Effect

- $S_{KL}(p, w) = \frac{\partial x_k(p,w)}{\partial p_L} + \frac{\partial x_k(p,w)}{\partial w} \cdot x_L(p, w)$

Indirect Utility Function

- $v(p, w) \equiv \max_{x \in B(p,w)} u(x)$
- Properties (assuming $u(\cdot)$ is continuous and represents locally nonsatiated preferences over $\mathbb{R}^m_+$):
  1. Homogeneous of degree 0 in (p, w)
  2. Strictly increasing in w, non-increasing in p_k
  3. Quasiconvex: $\{(p, w) \mid v(p, w) \leq \overline{v}\}$ is convex for all $\overline{v}$
  4. Continuous in (p, w)

Roy's Identity

- $x_k(p, w) = -\frac{\partial v(p,w)}{\partial p_k} \bigg/ \frac{\partial v(p,w)}{\partial w}$
- $\frac{\partial v(p,w)}{\partial p_k} = -x_k(p, w) \cdot \frac{\partial v(p,w)}{\partial w}$

Expenditure Minimization Problem

- $\min_{x_k \geq 0} \sum_{k=1}^{m} p_k x_k$ subject to $u(x) \geq \overline{u}$

Hicksian Demand

- $h(p, \overline{u})$: set of minimizers
- If $u(\cdot)$ is continuous and $\succeq$ is locally nonsatiated:
  1. Homogeneous of degree 0 in p
  2. Achieves utility exactly: $u(x) = \overline{u}$

Duality

- If $u(\cdot)$ is continuous and locally nonsatiated:
  1. $x(p, w) = h(p, v(p, w))$
  2. $h(p, \overline{u}) = x(p, e(p, \overline{u}))$
  3. $e(p, v(p, w)) = w$
  4. $v(p, e(p, \overline{u})) = \overline{u}$

Expenditure Function

- $e(p, \overline{u})$: minimal expenditure to achieve utility $\overline{u}$
- Properties (if $\succeq$ is continuous and locally nonsatiated):
  1. Homogeneous of degree 1 in p
  2. Strictly increasing in $\overline{u}$; non-decreasing in p_k
  3. Concave in p
  4. Continuous in $(p, \overline{u})$

Shepard's Lemma

- $h_k(p, \overline{u}) = \frac{\partial e(p, \overline{u})}{\partial p_k}$

Welfare Measures

- Compensating Variation: $CV(p^o, p', w) = w - e(p', u^o)$
- Equivalent Variation: $EV(p^o, p', w) = e(p^o, u') - w$
- $u' = v(p', w), \quad u^o = v(p^o, w)$

Profit Maximization

- $\max\limits_{z \in \mathbb{R}^{m-1}} pf(z) - wz$
- w: price of inputs z
- p: price of output $f(z)$
- $f(z)$: input demand function
- $q(p)$: supply function

Cost Minimization

- $\min_{z \in \mathbb{R}^{m-1}} p_{-m} z$ subject to $f(z) \geq q$
- $C(p_{-m}, q)$: cost function
- Profit: $\pi(p) = \max_{q \in \mathbb{R}_+} p_m q - c(p_{-m}, q)$
- Shepard's Lemma: $z_k(p_{-m}, q) = \frac{\partial c(p_{-m}, q)}{\partial p_k}$
- Properties of $c(p_{-m}, q)$:
  - Homogeneous of degree 1 in p_{-m}
  - Non-decreasing in q
  - Concave in p_{-m}